Shielding and charging of dust particles in the plasma sheath

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(Received 26 April 1999)

General physical features of the ion particle distribution function in the plasma sheath derived theoretically and observed experimentally are used for describing the dust shielding and charging in the sheath. It is shown that the shielding and charging of dust depend strongly on the degree of anisotropy of the ion distribution function and on the difference in the ion temperatures parallel and antiparallel with respect to the ion flow. Regions of ''antishielding'' appear both along the ion flow and perpendicular to it. The potential wells can be responsible for the attraction of other dust particles and for the creation of dust aggregates. This effect is related to the change of the sign of the dielectric function in the range of wave vectors corresponding to the strong Landau damping. [S1063-651X(99)14610-5]

PACS number(s): 52.35.Ra, 52.35.Kt, 52.25.Vy

I. INTRODUCTION

Most of the dusty plasma experiments [1,2] were performed in the sheath region close to the wall in a gas discharge plasma. The presence of the electric field in the sheath plays an important role in the dust trapping and levitation, since the dust particles receiving a large charge by plasma currents are strongly affected by the electric field of the sheath, which acts against the ion drag and gravity forces and allows the dust to levitate. The latter phenomenon serves as a basic process for the formation of dust clouds in rf etching plasma [3,4] and the formation of dust crystals [1,2,5,6].

According to the Bohm criterion [7], plasma ions, accelerated by the sheath electric field, can achieve supersonic velocities in the plasma sheath. Such ion flows could excite longitudinal plasma oscillations in the sheath region, and is well known both from numerical simulations and experimental observations [8,9]. The influence of the longitudinal random fields leads to the increased broadening of the ion distribution in the direction of the ion flow. In this case, the width of the ion distribution in the direction parallel to the ion flow (''downstream'') should be larger than that in the perpendicular direction. Thus the ion drifting distribution is highly anisotropic. Moreover, the downstream parallel ion distribution should be wider than the ion distribution in the opposite direction (''upstream''). This is due to the fact that the oscillations excited upstream act mostly on the downstream particles. It is noted also that the electric field of the sheath self-consistently produces an ion distribution downstream-upstream asymmetry in widths [10]. Both arguments (the excited plasma turbulent fields and the influence of the electric field on the ion distribution) suggest that the ion distribution in the sheath should be asymmetric. This characteristic feature must be taken into account when considering charging and shielding of dust particles.

So far there have been few experimental measurements of the ion distribution in the sheath. The most reliable is the recent investigation of the ion velocity distribution in the sheath by means of a laser induced fluorescence [11]. These experiments show that the general observed features of the ion distribution asymmetry agree qualitatively with the theoretical predictions [10] in which the effect of the excited turbulent plasma fields in the sheath was neglected. However, the measured absolute value of the width appears to be much larger than the theoretical result [10]. A plausible candidate to explain this anomalous width is the sheath turbulence. We stress that the turbulence also modifies the regular electric field in the sheath and can therefore affect the results of [10] in this way.

The important observation of the previous experiment [11] is that the observed width of the ion distribution perpendicular to the flow is at least one order of magnitude less than the upstream (or downstream) width. The ion flow can thus finally appear to be moderate as compared with the thermal ion velocity parallel to the drift. The ion thermal velocity in the direction perpendicular to the ion drift is almost the same as in the bulk plasma and is therefore small compared with the ion flow velocity. The collective field that can broaden the ion distribution can be created mainly in the direction of ion flow. For the moderate longitudinal ion flow the problem of shielding can be considered only numerically since the Landau damping of the fields of the dust particle

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moving through the ion plasma (in the frame of ions at rest) should be rather large.

In this paper, we propose the theory of shielding and charging of dust in the plasma sheath, using the ion distribution function with a marginal (subsonic) velocity, which is of the order of the characteristic broadening of the ion distribution in the longitudinal direction. In this framework, the physics of shielding and charging is strongly modified as compared to the case of superthermal (and especially supersonic) ion flows. The main mechanism in our case is closely linked to the Landau damping of collective plasma perturbations excited by flowing ions.

We will not discuss the details of the mechanism that produces the ion distribution in the sheath. The main subject of the present paper is the investigation of the dust charging and shielding for the ion distributions often present in the sheath. Thus we use the above discussed theoretical and experimental results [10,11] to construct a model for the shielding of the dust particle field.

In the presence of the supersonic ion flow, dust particles excite the wake sound field in which the shielding can be changed to antishielding; since the wake wave field changes periodically, the sign of the plasma charge accumulation is also changed [12,13]. The dust particles in this case (of the supersonic ion flow) create a potential well for neighboring dust particles. The generation of the wake by stationary dust grains is physically equivalent to the Cherenkov wave emission in the frame of ions at rest [14]. Note that the shielding of the dust particle field can also be nonlinear due to large dust charges [15]. Similarly, we expect that the dust particle can excite a nonlinear wake. The problem of nonlinear excitation of the wake field by a planar charged sheath can have an exact solution [16], but the three dimensional nonlinear theory of the wake field excitation is much more complicated as known from the theory of the wake field accelerators [17].

In this paper, we deal with the case of much lower (subsonic) drift velocities, which is physically different from the wake excitation by a supersonic ion flow, as the wake field cannot be excited in this case. We show that, in this case, the stopping power of ions by the Landau mechanism changes the ion distribution around the dust particle and produces the ion bunching as well as the local excess of positive charges around the dust particle. The latter can create potential wells not only in the direction parallel to the ion flow but also in the direction that is almost perpendicular to it (note that the wake field has the analogous behavior [13]). Nonlinear corrections to the linear shielding are also discussed in this paper.

The outline of the paper follows. In Sec. II, the model of the ion distribution is formulated and the dielectric permittivity used for investigating the shielding of dust in the plasma sheath is derived. Section III deals with the dust particle shielding. In Sec. IV, the dust charging in the sheath is investigated. In Sec. V, numerical results for the dust shielding and charging in the sheath are presented. Section VI is devoted to the discussion of the role of nonlinear effects in the charge screening. Conclusions are summarized in Sec. VII.

II. MODEL OF ION DISTRIBUTION AND THE DIELECTRIC PERMITTIVITY

The ion distribution function f_i we use is characterized by three temperatures: T_i is the temperature perpendicular to the direction of the ion flow, T_1 is the temperature that corresponds to the direction parallel to the ion flow (which we call the downstream direction), and T_2 is the temperature in the direction antiparallel to the ion flow (which we call the upstream direction). Thus the ion distribution function can be written as

$$f_{i(1,2)}=\frac{n_i}{(2\pi)^{3/2}v_{Ti}^2 v_T}\exp\left(-\frac{v_\perp^2}{2v_{Ti}^2}-\frac{(v-u)^2}{2v_{(T1,T2)}^2}\right), \qquad (1)$$

where u is the ion flow velocity, v corresponds to the ion velocity component along the drift direction, and $v_\perp$ corresponds to the velocity component perpendicular to the drift; the subscript 1 corresponds to the downstream part of the ion distribution where $v>u$ and the subscript 2 corresponds to the upstream part of the distribution function where $v<u$; $v_{Ti}\equiv\sqrt{T_i/m_i}$ is the thermal ion velocity characterizing the width of the ion distribution perpendicular to the drift, v_{T1} and v_{T2} are the ion downstream and upstream thermal velocities, characterizing the width of the distribution function downstream and upstream in the direction of the drift, respectively. Finally, $v_T=(v_{T1}+v_{T2})/2$ is the averaged thermal velocity along the ion drift. We assume

$$T_i \ll T_2 < T_1. \qquad (2)$$

The linear dielectric function ε of singly ionized ions for this distribution can be written in the form

$$\varepsilon=1+\frac{1}{k^2 d_i^2}+\frac{4\pi e^2}{k^2}\int\frac{1}{k(u+v)+\mathbf{k}_\perp\cdot\mathbf{v}_\perp}\times\left[\frac{kv}{v_{T1,T2}^2}-\frac{k(u+v)}{v_{Ti}^2}\right]f_i^r\,d^3v, \qquad (3)$$

where k and v are the components of the wave vector and ion velocity along the direction of the drift, respectively, while $\mathbf{k}_\perp$ and the $\mathbf{v}_\perp$ are the components perpendicular to it. The f_i^r is the ion distribution in the frame of ions at rest and, finally, $d_i=v_{Ti}/\omega_{pi}$ (where ω_{pi} is the ion plasma frequency) is the ion Debye length determined by the ion perpendicular thermal velocity. It is noted that, in the frame of ions at rest, the plasma is moving with the velocity $-u$ and thus only the upstream part of the ion distribution can produce the strong Landau damping.

In order to obtain the analytical expression for the dielectric function, we further simplify Eq. (3) by using inequalities (2). For $k_\perp/k \gg v_{T1,T2}/v_{Ti} \gg 1$ the wave vectors are mainly in the plane perpendicular to the ion flow. This case corresponds to the shielding field with $r_\perp/r \ll v_{Ti}/v_{T1,T2}$, i.e., in the directions at small angles with respect to the ion flow. In this narrow angular interval we can neglect the contribution $k(u+v)$ in the denominator of the third term of Eq. (3) and the integration over the angles ϕ perpendicular to the plane of the ion drift gives $\int d\phi/\mathbf{k}_\perp\cdot\mathbf{v}_\perp \approx -i\pi\int\delta(\mathbf{k}_\perp\cdot\mathbf{v}_\perp)$. We therefore find for the last term the following estimate for the expression in front of f_i^r:

$$\frac{\pi k v_{T1,T2}}{v_{Ti}^2 k_\perp v_{Ti}} \ll \frac{1}{v_{Ti}^2},$$

which means that the last term in ε can be neglected. Thus we approximately have

$$\varepsilon \approx 1 + \frac{1}{k^2 d_i^2}. \tag{4}$$

Thus within the narrow interval of angles θ (i.e., mainly along the ion flow),

$$\theta \ll \frac{v_{Ti}}{v_{T1,T2}}, \tag{5}$$

the shielding is of Debye character with the perpendicular ion temperature. Condition (5) is the condition when such a screening can take place. In the sheath region, limitation (5) appears to be rather strong if indeed the ion perpendicular temperature is much smaller than the longitudinal one.

Outside the interval given by inequality (5), we can neglect the term $\mathbf{k}_\perp \cdot \mathbf{v}_\perp$ in the denominator of the last term in ε and the second term in the brackets of the last term canceled by the term $1/k^2 d_i^2$, thus yielding ε as

$$\varepsilon \approx 1 + \frac{4\pi e^2}{k^2} \int \frac{kv}{k(v+u) v_{T1,T2}^2} f_i^r d^3 v. \tag{6}$$

Then the screening and its modification is determined by the longitudinal ion distributions. We can introduce the Debye length for the ''averaged'' longitudinal temperature

$$d^2 \equiv \frac{[(\sqrt{T_1} + \sqrt{T_2})/2]^2}{4\pi n_i e^2} \tag{7}$$

and the relative flow velocity

$$w_1 = \frac{u}{\sqrt{2} v_{T1}}; \quad w_2 = \frac{u}{\sqrt{2} v_{T2}} = \frac{1}{\sqrt{t}} w, \tag{8}$$

where

$$w = u/\sqrt{2} v_{T1}, \quad t \equiv \frac{T_2}{T_1}. \tag{9}$$

We also introduce here other dimensionless parameters which are used in the paper:

$$\tau = \frac{T_i}{T_e}, \quad \tau_1 = \frac{T_i}{T_1}, \quad z = \frac{Z_d e^2}{a T_e}. \tag{10}$$

The dielectric function for the screening at large angles to the ion flow can be written in the form

$$\varepsilon = 1 + \frac{1}{k^2 d^2} \frac{1+\sqrt{t}}{2\sqrt{\pi}} \int_0^\infty e^{-y^2} y\, dy \times \left(\frac{1}{y + w_1 - i0} + \frac{1}{\sqrt{t}(y - w_2 + i0)} \right). \tag{11}$$

Here we define u as being positive, then we have $w_1 > 0$, $w_2 > 0$, and the imaginary part of Eq. (11) can be expressed as usual in the analytical form. We write the dielectric permittivity ε as

$$\varepsilon = 1 + \frac{1}{k^2 d^2} W(w), \tag{12}$$

where

$$W(w) = W_R(w) - i \frac{k}{|k|} W_I(w). \tag{13}$$

We have

$$W_R(w) = \frac{1+\sqrt{t}}{2\sqrt{\pi}} P \int_0^\infty e^{-y^2} y\, dy \left(\frac{1}{y+w} + \frac{1}{\sqrt{t} y - w} \right) \tag{14}$$

and

$$W_I(w) = \frac{\sqrt{\pi}}{2} \left(1 + \frac{1}{\sqrt{t}} \right) \frac{w}{\sqrt{t}} e^{-w^2/t}. \tag{15}$$

The negative sign in the imaginary part of Eq. (13) shows that the plasma perturbations propagating along the direction of ion flow are amplified, while the perturbations propagating against the direction of the flow are damped. Both damping and amplification are strong for the moderate drift (w of order of 1) and cannot be described as the wave excitation, since the flow velocity is subsonic. This effect is the manifestation of the strong Landau damping (since the amplification is just the inversed Landau damping).

III. DUST PARTICLE SHIELDING

The potential of a dust particle is normalized to its Coulomb potential $-Z_d e/r$ and can be written as

$$\phi = \frac{r}{2\pi^2} \mathrm{Re} \int \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2 \varepsilon} d^3 k. \tag{16}$$

The presence of $k/|k|$ in the dielectric permittivity ε changes the screened potential in the upstream and downstream directions. The $\mathbf{r}$ is the direction we are considering for screening. Let us denote the component of the wave vector along this direction as k'. This component is equal to k for the downstream direction and $-k$ in the upstream direction. This can be seen from introducing the spherical or cylindrical coordinates in $\mathbf{r}$ space where the drift will be directed along the z axis for the downstream direction and negative z axis for the upstream direction. The potential ϕ_+ represents the downstream potential, while the potential in the upstream direction is ϕ_-. Introducing $y = k_\perp r$, $x = k' r$ ($k'_\perp = k_\perp$) and $r = r/d$, we find

$$\phi_\pm = \frac{2}{\pi} \int_0^\infty dx \int_0^\infty \frac{J_0(y \sin\theta) y\, dy}{(y^2 + x^2 + r^2 W_R)^2 + (r^2 W_I)^2} \times [(y^2 + x^2 + r^2 W_R) \cos(x\cos\theta) \mp W_I r^2 \sin(x \cos\theta)]. \tag{17}$$

This expression is useful to find the distribution of the potential perpendicular to the direction of the flow

$$\phi_{\perp}=\frac{2}{\pi}\int_0^{\infty}dx\int_0^{\infty}\frac{J_0(y)y\,dy}{(y^2+x^2+r^2W_R)^2+(r^2W_I)^2}\times(y^2+x^2+r^2W_R). \tag{18}$$

Integrating in x is easily performed by converting the integration interval to $[-\infty,+\infty]$ and determining the residues at pole integration in the complex x plane.

For the directions along the drift it is more convenient to use the spherical coordinates and integrate over the angles of the wave vector

$$\phi_{\pm}=\frac{2}{\pi}\int_0^{\infty}\frac{\sin y(y^2+r^2W_R)y\,dy}{(y^2+r^2W_R)^2+(r^2W_I)^2}\mp\frac{4}{\pi}r^2W_I\int_0^{\infty}\frac{\sin^2(y/2)y\,dy}{(y^2+r^2W_R)^2+(r^2W_I)^2}. \tag{19}$$

The first integral in Eq. (19) can be computed by the pole residue method in the complex plane, while the second one should be calculated numerically. Let us note that the first term of Eq. (19) reduces to 1 in the case where one neglects the shielding and Landau damping, and the second term is completely determined by the Landau damping.

The condition of the validity of Eqs. (17) and (18) is $r_{\perp}/|z|\gg v_{Ti}/v_{T1,T2}$, which defines the corresponding solid angle. Thus the validity domain of Eq. (19) is outside the cone defined by this solid angle. Since the latter is small, we can use the approximation that this validity domain contains directions close to the ion flow direction. This consideration can be made only because of the presence of the small parameter $v_{Ti}/v_{T1,T2}$. Note that the linear approximation for the screening of dust particles is only the first step in considering the problem, since nonlinearities in the screening could be significant as it is discussed below in Sec. VI.

IV. DUST CHARGING IN THE SHEATH

In the sheath the charge neutrality is violated and, therefore, the ion and electron densities are not equal. We use the parameter $P=(n_i-n_e)/n_e$ to characterize the difference between the ion density n_i and the electron density n_e; in the sheath we have $P>0$. We remind that the ion distribution function f_i is characterized by the three temperatures T_i, T_1, and T_2, and is given by Eq. (1).

In the limits of the orbit limited motion approximation, the ion (and, respectively, electron) currents on a dust particle are given by (for singly ionized ions)

$$J_{i,e}=\pm e\pi a^2\int\sqrt{v^2+v_{\perp}^2}\left(1\pm\frac{2Z_de^2}{m_{i,e}(v^2+v_{\perp}^2)}\right)f_{i,e}\,d^3v, \tag{20}$$

where a is the radius of the spherical dust particle and the $+$ (respectively, $-$) sign corresponds to ions (electrons). We express J_i in the form useful for numerical computations by introducing dimensionless parameters (10). According to experimental results, $\tau_1\ll 1$ and $t<1$. We write down the simplified result for the ion current in the limit $z/\tau\gg 1$:

$$J_i=\frac{2\sqrt{2}}{\sqrt{\pi}}\frac{z}{\tau}en_i(x)v_{Ti}\pi a^2G_i, \tag{21}$$

where G_i is given by

$$G_i(w,t,r)=\frac{\sqrt{\pi}}{1+\sqrt{t}}w\int_0^{\infty}e^{-w^2x^2}dx\left\{\exp\left(\frac{w^2(x+1)^2}{\tau_1}\right)\times\left[1-\operatorname{erf}\left(\frac{w(x+1)}{\sqrt{\tau_1}}\right)\right]\right\}+\frac{\sqrt{\pi}t}{1+\sqrt{t}}w\times\int_0^{\infty}e^{-w^2x^2}dx\left\{\exp\left(\frac{w^2(x-1)^2t}{\tau}\right)\times\left[1-\operatorname{erf}\left(\frac{\sqrt{t}|x-1|w}{\sqrt{t}}\right)\right]\right\}, \tag{22}$$

and $\operatorname{erf}(x)$ is the error function

$$\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x\exp(-x^2)dx. \tag{23}$$

From Eq. (20), we obtain for the electron current

$$J_e=-\frac{2\sqrt{2}}{\sqrt{\pi}}e\pi a^2v_{Te}n_e\exp(-z), \tag{24}$$

where $v_{Te}=(T_e/m_e)^{1/2}$. The equilibrium charge of the dust particle can be found from the condition of the zero total current, which leads to the equation

$$G_e=\frac{n_i}{n_e}\frac{1}{\sqrt{\mu\tau}}zG_i=\frac{1+P}{\sqrt{\mu\tau}}zG_i, \tag{25}$$

where $\mu=m_i/m_e$ is the electron to ion mass ratio.

V. NUMERICAL RESULTS

In order to investigate the dust shielding, we have numerically calculated the real and imaginary parts of the dielectric function Eq. (3). Figure 1 shows that the negative values of the dielectric function will appear mostly in the range of the strong Landau damping and will be present only for the moderate drift velocities. The appearance of the damping implies the existence of the collective stopping power for the incoming ions, which allows an additional accumulation of the ion charges around the dust particle.

The usual process of screening in the absence of the flow ends when the charge of the particle is totally compensated by the ambient plasma. The Landau damping provides corrections, which are of the order of w for $w\ll 1$, as defined in Eq. (9). For w of order 1, the stopping power related to the Landau damping is appreciable and the incoming ions from the drift flow can create a total positive polarization charge around the dust particle. This charge can be larger than the negative charge of the dust particle itself leading to the attraction of other negatively charged dust particles, and results in creating the potential well.

The potential (normalized to the nonscreened Coulomb

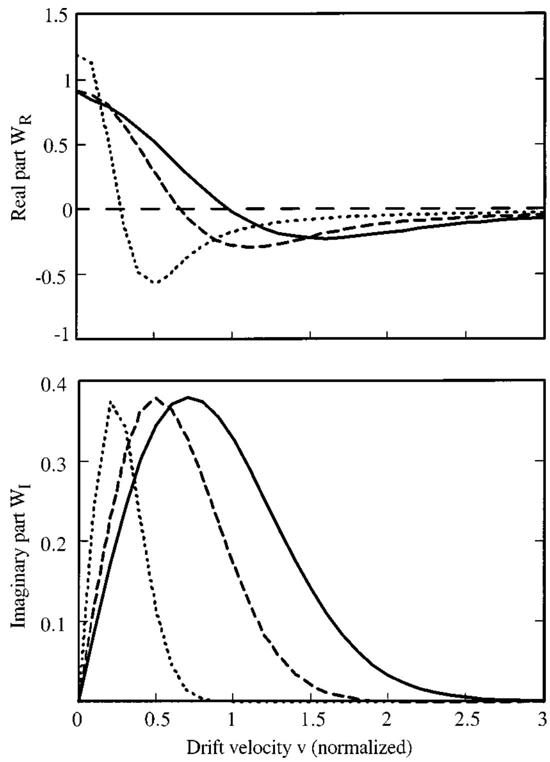

FIG. 1. Real and imaginary parts of the dielectric function. The solid curves correspond to $t=1$, the dashed curves correspond to $t=\frac{1}{2}$, and the dotted curves correspond to $t=\frac{1}{10}$.

potential) of the dust particle in the perpendicular direction to the ion flow is shown in Fig. 2. The normalized potential is presented as a function of the ratio of the distance from the dust particle to the Debye length d defined by Eq. (7). One can see that the potential well indeed exists and increases with the increasing w and/or decreasing t. Figure 3(a) shows

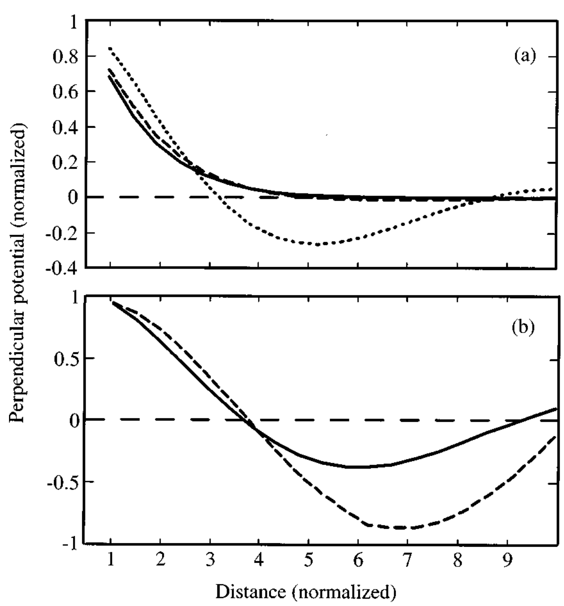

FIG. 2. (a) Potential well for $t=\frac{1}{2}$ and different values of w: $w=0$, dashed dotted curve; $w=\frac{1}{2}$, dotted curve; $w=1$, dashed curve; $w=\frac{3}{2}$, solid curve. (b) The potential well for $w=1$; the solid curve corresponds to $t=1$, the dashed curve corresponds to $t=\frac{1}{2}$, and the dotted curve corresponds to $t=\frac{1}{5}$.

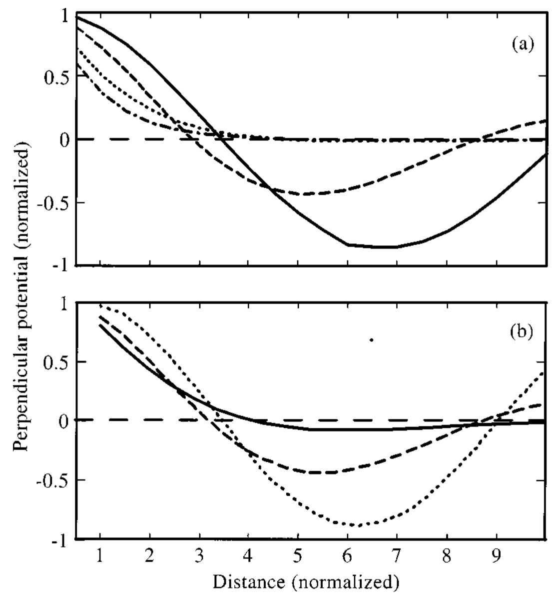

FIG. 3. (a) Same as in Fig. 2(b) for $w=\frac{1}{2}$; the solid curve corresponds to $t=1$, the dashed curve corresponds to $t=\frac{1}{2}$, and the dotted curve corresponds to $t=\frac{1}{5}$. (b) Similar to (a) for $w=\frac{3}{2}$; $t=1$ (solid curve) and $t=\frac{1}{2}$ (dashed curve).

the same behavior for $w=\frac{1}{2}$. Here the potential well is almost nonexistent for $t=1$, it is quite small for $t=\frac{1}{2}$, and becomes appreciable for $t=\frac{1}{5}$. Figure 3(b) shows the results for higher velocities of the flow $w=\frac{3}{2}$. In this case the potential well is deep and shifted to higher r, again increasing with decreasing t. For $t=1$ (simple drift with anisotropy of the temperature), the potential well exists only for $u>1$.

We also analyzed expressions for the potentials created by the dust particle close to the upstream and downstream directions, but for outside the narrow cone where the Debye shielding is present. In principle, this cone narrows with decreasing v_{Ti}, and the expressions used are approximately valid for the almost upstream and downstream directions. Figure 4(a) shows the results of numerical calculations for the downstream case for $t=1/2$, $w=1/2$, and $w=1$. We can see that the potential well increases with the increasing flow velocity. The tendency for the upstream case shown in Fig. 4(b) is the opposite; with the increase of the ion flow velocity, the dust particle becomes more shielded and the potential well never appears. This has a simple explanation since the stopping power in the direction opposite to the ion flow is not sufficient to create significant ion space charges to change the sign of the shielding cloud. For comparison, the Debye shielding potentials for these cases are also plotted. Figure 5 illustrates the fact that, the smaller the angle is between the direction of the flow and the direction of the screening, the deeper the potential well is and the larger the distance between the dust particle and the position of the potential minimum.

Finally, Fig. 6 shows the numerical solutions for the equilibrium dust charges for singly ionized argon gas, where $\mu=1860\times 40$ and $\tau=3\times 10^{-2}$. The dust charge increases both with the increasing flow velocity w and the (inverse) asymmetry $1/t$. This is mainly due to the decrease in the cross section of the interaction of ions with dust particles whenever the flow velocity is increased. This leads to the

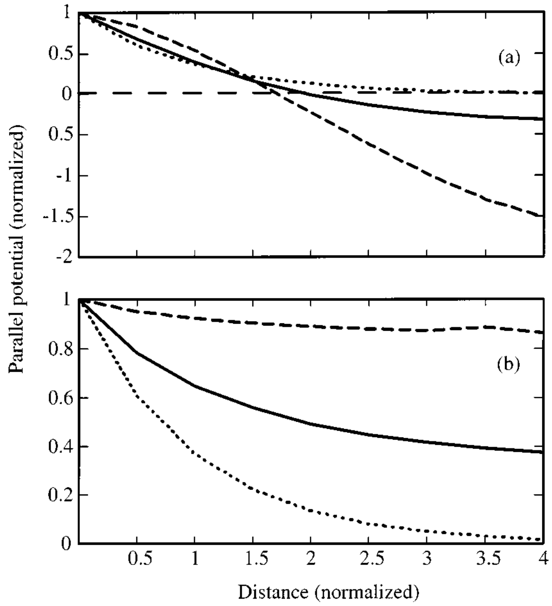

FIG. 4. (a) Dependence of the normalized dust potential upon the distance from the dust particle for the case of the direction close to the downstream direction. The solid curve is obtained for $w=\frac{1}{2}$, the dashed curve, for $w=1$, and the dotted curve presents the exponential Debye screening. All of the results are obtained for $t=\frac{1}{3}$. (b) The same as in (a), but in the upstream direction.

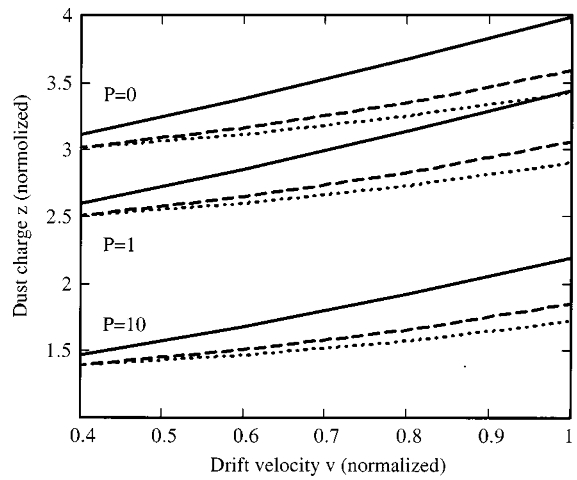

FIG. 6. Dependence of the normalized charge z on the speed of the ion flow for various values of the parameter P (measuring the degree of quasineutrality in the system). The solid lines correspond to $t=\frac{1}{5}$, the dashed lines correspond to $t=\frac{3}{5}$, and the dotted lines correspond to $t=1$.

decrease in the ion current and the greater charging of dust particles by the electron current. We also found that, in the case of equal downstream and upstream temperatures, the dust charges are smaller than in the case when the downstream temperature is larger than the upstream one.

VI. DISCUSSION

Let us first comment on the relation between the wake field potential wells and the Landau potential well. In the case of the large (supersonic) ion flow velocity, the dust particle velocity in the frame of ions at rest substantially exceeds all of the ion thermal velocities (along and perpendicular to the drift). Then the Cherenkov condition for the excitation of plasma modes by dust is fulfilled (in this case, such modes exist in the ion plasma at rest) and the emission of these modes as a wake is possible [12,13]. For the moderate subsonic longitudinal ion drifts, which are the subject of the present study, the presence of the strong Landau damping is important since there are no excited wake fields. In the case of the strong Landau damping, the potential wells are mostly concentrated in the range of wave vectors belonging to the near zone where the electric field produced by the dust particle is of dissipative character (but not the wave field, as in the case of the wake excitation). The creation of this field is independent of whether the wake field is created (since it is in the near zone) and is even independent as to whether the ion plasma can have weakly damped modes.

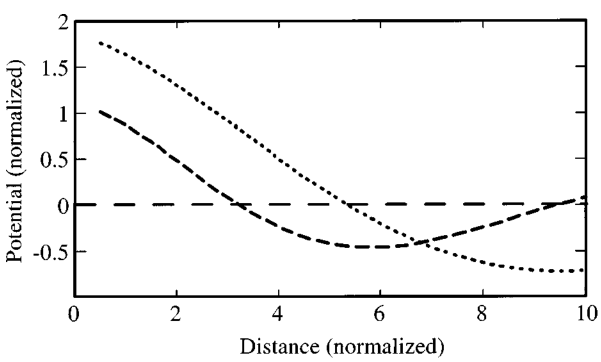

FIG. 5. Dependence of the dust potential (normalized to the Coulomb potential) on the angle with respect to the ion flow (such that $\Theta=\pi/2$, corresponds to the downstream direction). The dashed curve is for $\Theta=3\pi/10$ and the dotted curve is for $\Theta=2\pi/5$.

The role of nonlinearities produced by the dust particles field is an important issue. Without detailed calculations, we present here the dimensional analysis, which can give the order of magnitude of the distances where the plasma polarization charge can change and screen the dust charge independently of the sign of this polarization charge. Within the narrow angular interval with respect to the flow, where Eq. (4) is valid, the characteristic distance is of order d_i and for other angles it is determined by Eq. (7). Note that for the isotropic ion distribution, the only characteristic length is d_i. Nonlinear effects are always strong close to the dust grain. Indeed, for the isotropic case, the condition for the nonlinearity to be weak on the dust surface is

$$\frac{Z_d e^2}{a T_i}=\frac{T_e}{T_i} z \ll 1, \tag{26}$$

where a is the size of dust particle, T_e and T_i are the electron and ion temperatures, and $z \equiv Z_d e^2/a T_e$ is the dimensionless dust charge, which is usually of the order 2. Since in most of the sheath experiments the ratio T_e/T_i is high $\approx 10^2$, inequality (26) can never be satisfied.

However, the linear approximation can correctly describe the shielding if the nonlinearities are at least weak at the shielding radius. For the ion distribution (1), the calculation of the ratio of the nonlinear charge density $\rho_{\mathbf{k}}^N$ to the linear charge density $\rho_{\mathbf{k}}^L=-Z_d e\,\delta(\omega+ku)/2\pi^2$ of a dust particle in the frame of ions at rest gives, for $k\sim k_\perp$,

$$\Delta_{\mathbf{k}}=\frac{\rho_{\mathbf{k}}^{N}}{\rho_{\mathbf{k}}^{L}}=\frac{Z_d w}{\sqrt{\pi}(4\pi)^2 n d_i^3 k_\perp d} I(w,t), \tag{27}$$

where

$$I(w,t)=\int_0^\infty dx \int_0^\infty ds \frac{x^2 s^4}{|x^2 s^2+s^2+4x^2 W(w,t)/(1+\sqrt{t})^2|^2} \times\left[\int_0^\infty y\,dy\,\exp[-y^2-s^2(y-w)^2/x^2] - \sqrt{t}\int_0^\infty y\,dy\,\exp[-(y^2/t)-s^2(y+w)^2/x^2]\right]. \tag{28}$$

Numerical calculations give $I(\frac{1}{2},\frac{1}{2})=0.819$ and $I(1,\frac{1}{2})=1.362$. The condition that the nonlinearity in the direction of the ion flow is small (with $k_\perp \sim 2\pi/r$) thus becomes

$$Z_d \ll \alpha(2\pi)^2 n_i d_i^3 \frac{d}{r}, \tag{29}$$

where $\alpha \approx 5$ for $w=\frac{1}{2}$ and $t=\frac{1}{2}$, and $\alpha \approx 1.5$ for $w=1$ and $t=\frac{1}{2}$. For $r\approx 6d$ (which is the approximate position of the minimum of the potential well, according to the numerical results), $n_i \approx 2\times 10^9\ \mathrm{cm}^{-3}$ and $d_i \approx 5\times 10^{-3}$ cm. Then the right hand side of Eq. (29) is of the order 10^3, which can correspond to the observed dust charges for not too large dust particles.

The results given in Sec. V can thus be considered as (to the best of our knowledge) the first approach to the problem, since nonlinearities in the dust particle shielding could be important, especially for larger particles. We also note that the linear approach can still have a range of applicability in the sheath where the ion flow is mainly regulated by the potential in the sheath. The change of the ion flow produced by a single dust particle is small for the dust particles of small size, as compared to the size of the wall producing the sheath. However, this statement is incorrect if the distance between the dust particle and the position of the minimum of the potential well is much less than the distance between the dust particle and the wall (electrode). For a more detailed description it is necessary to take into account the dependence of the ion flow velocity on the position of the dust particle with respect to the wall. This effect can be especially important in the case when the flow velocity at the position of the dust particle significantly differs from the velocity of the flow at the wall.

VII. CONCLUSION

Using a realistic model for the ion distribution in the plasma sheath inferred from the theoretical expectations and the experimental observations, we have investigated the dust shielding and charging in the sheath. It is shown that the latter processes depend strongly on the degree of anisotropy of the ion distribution, as well as on the difference between the downstream and upstream ion longitudinal temperatures.

The physical mechanism responsible for the appearance of the additional ion charge accumulation close to the dust particle is connected with the strong Landau damping. This is an important result of this investigation. However, the question of the effects of plasma nonlinearities in the linearly derived potential well remains open. If the potential well is created and another dust particle appears to be at the position of this well, it will strongly absorb the additional ions and decrease the value of the well. These problems can be further investigated only by appropriate numerical simulations.

ACKNOWLEDGMENTS

V.N.T. and S.V.V. wish to thank Equipe Dynamique des Systèmes Complexes of CNRS–Université de Provence for support and hospitality during their visit. The work of S.V.V. was partially supported by the Australian Research Council.

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